DISTRIBUTION OF MEAN TEMPERATURES OF A POROUS MATERIAL AND COOLANT IN A CHANNEL WITH A POROUS INSERT

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Expressions are obtained to determine the mean temperatures of a porous material and coolant and an analysis is made of the character of the temperature distribution along a channel with a porous insert.

One efficient method of cooling the walls of structural elements of power plants in the case of very high heat fluxes is the installation of porous metallic inserts in the channel of the cooling system, with the insert being securely fastened to the channel walls. In this case, heat is transferred from the walls of the channel by conduction over the frame of porous metal and from the frame to the coolant by intrapore heat transfer.

A physical model of such a process for the motion of a single-phase coolant in a channel with a porous, high-heat-conducting insert was described in detail in [1-3]. In [1] a numerical method was used to find the distribution of the temperatures of the porous metal and coolant for certain particular parameters of the process. In [2] an analytical expression was derived for calculation of the rate of heat transfer between the wall of a planar channel and the coolant, and the effect of the temperature difference $T - t$ between the porous metal and coolant on this rate was determined. The solution was obtained for the region of stabilized heat transfer, which was distant from the ends of the channel. It was assumed that the local t and mean $\bar{t}$ temperatures of the coolant also change linearly in the presence of heat conduction along the channel axis and that the gradient of the mean temperature $\partial\bar{t}/\partial x$ of the coolant along the channel axis coincides with the temperature gradient along the same axis at any point of the given section of the channel: $\partial\bar{t}/\partial x = \partial t/\partial x = \text{const}$, which is valid, as will be shown below, if there are low-heat-conducting inserts in the channel.

The study [3] developed a method of determining the thermal characteristics of channels filled with porous metals. Measurements were made of the heat-transfer coefficient and thermal conductivity of the channel when filled with porous sintered copper. Test data on the rate of intrapore heat transfer for air and helium was generalized by the criterional relation $\mathrm{Nu}_V = 0.1\mathrm{Pe}$. It should be noted that in [3] the solution of the problem of axial heat flow was approached on the basis of the assumption of an S-shaped longitudinal temperature profile. Linearity was assumed only beyond the central section of the channel, the length of which depends on the parameters of the heat-transfer process.

The goal of the present study is to derive analytical expressions for calculating the field of the temperatures of a porous material and a single-phase coolant during motion of the coolant along the channel. The temperatures are averaged over the cross section of the circular annular channel. Another goal is to numerically study the effect of different factors of the process on the character of change of the mean temperatures of the porous material and coolant along the channel.

Remaining within the framework of the physical model of the motion of coolant in a channel with a porous insert constructed in [1-3], we write the equations determining the temperature field of the porous material and coolant for its flow in a circular annular channel with a porous insert:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) = h_v(T - t), \qquad (1)$$

$$Gc\frac{\partial t}{\partial x} = h_v(T - t)$$

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with the following boundary conditions:

$$x = 0,\ t = t_0;\ \lambda \frac{\partial T}{\partial x} = 0;$$
$$x = l,\ \lambda \frac{\partial T}{\partial x} = 0;$$
$$r = R_1,\ -\lambda \frac{\partial T}{\partial r} = q_1(x);$$
$$r = R_2,\ \lambda \frac{\partial T}{\partial r} = q_2(x). \tag{2}$$

Let us average Eqs. (1) over the cross section of the channel. This gives us the following system of equations:

$$2\frac{R_1 q_1(x) + R_2 q_2(x)}{R_2^2 - R_1^2} + \lambda \frac{d^2\overline{T}}{dx^2} = h_v(\overline{T} - \overline{t}), \tag{3}$$
$$Gc\frac{d\overline{t}}{dx} = h_v(\overline{T} - \overline{t})$$

with the boundary conditions:

$$x = 0,\ \overline{t} = t_0;\ \lambda \frac{d\overline{T}}{dx} = 0;\ x = l,\ \lambda \frac{d\overline{T}}{dx} = 0, \tag{4}$$

where

$$\overline{T} = \frac{2\pi}{R_2^2 - R_1^2}\int\limits_{R_1}^{R_2} rT dr;\ \overline{t} = \frac{2\pi}{R_2^2 - R_1^2}\int\limits_{R_1}^{R_2} rt dr.$$

System (3), with boundary conditions (4), describes the field of the temperatures of the porous material $\overline{T}$ and the single-phase coolant $\overline{t}$ during coolant motion in the channel when the temperatures are averaged over the cross section of a circular annular channel with a porous insert.

It follows from system (3) that

$$\lambda \frac{d^2\overline{T}}{dx^2} + 2\frac{R_1 q_1(x) + R_2 q_2(x)}{R_2^2 - R_1^2} = Gc\frac{d\overline{t}}{dx}. \tag{5}$$

Integrating (5) once over x from x = 0 to a certain running value of x, with allowance for conditions (4) we obtain the following expression:

$$\overline{t} = t_0 + \frac{\lambda}{Gc}\frac{d\overline{T}}{dx} + \frac{2}{Gc(R_2^2 - R_1^2)}\int\limits_0^x [R_1 q_1(x) + R_2 q_2(x)]\, dx. \tag{6}$$

Inserting (6) into the first equation of system (3), we obtain an equation to determine the mean temperature of the porous material along the channel containing the porous insert. With $q_2(x) = 0$ and $q_1(x) = q = \text{const}$, this equation can be written as follows:

$$\frac{d^2\overline{T}}{dx^2} + \frac{h_v}{Gc}\frac{d\overline{T}}{dx} - \frac{h_v}{\lambda}\overline{T} = -\left[\frac{h_v t_0}{\lambda} + \frac{2R_1 q}{\lambda(R_2^2 - R_1^2)}\right] - \frac{2h_v R_1 q}{Gc\lambda(R_2^2 - R_1^2)}x. \tag{7}$$

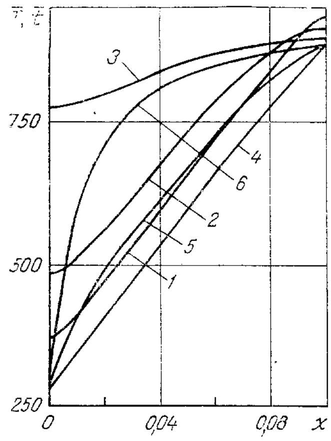

Fig. 1. Distribution of mean temperatures of the porous material and coolant along the channel. $\overline{T}$, $\overline{t}$, °K; x, m

Integrating linear inhomogeneous differential equation (7) and allowing for boundary conditions (4), we obtain the following analytic expression for the distribution of the temperature of the porous material along a circular annular channel with a porous insert when the temperature is averaged over the channel cross section:

$$\overline{T} = C_1 \exp(k_1 x) + C_2 \exp(k_2 x) + \overline{T}_0, \tag{8}$$

where

$$\overline{T}_0 = t_0 + \frac{2\lambda}{(Gc)^2}\frac{R_1 q}{R_2^2 - R_1^2} + \frac{2}{h_v}\frac{R_1 q}{R_2^2 - R_1^2} + \frac{2}{Gc}\frac{R_1 q}{R_2^2 - R_1^2}x;$$

$$C_1 = \frac{2}{Gc}\frac{R_1 q}{R_2^2 - R_1^2}\frac{1}{k_1}\frac{\exp(k_2 l) - 1}{\exp(k_1 l) - \exp(k_2 l)};$$

$$C_2 = \frac{2}{Gc}\frac{R_1 q}{R_2^2 - R_1^2}\frac{1}{k_2}\frac{\exp(k_1 l) - 1}{\exp(k_2 l) - \exp(k_1 l)};$$

$$k_1 = -\frac{h_v}{2Gc} + \sqrt{\left(\frac{h_v}{2Gc}\right)^2 + \frac{h_v}{\lambda}};$$

$$k_2 = -\frac{h_v}{2Gc} - \sqrt{\left(\frac{h_v}{2Gc}\right)^2 + \frac{h_v}{\lambda}}.$$

Inserting (8) into Eq. (6), we obtain the following analytic expression to find the distribution of the temperature of the coolant along such a channel when it is averaged across the channel:

$$\overline{t} = t_0 + \frac{2\lambda}{(Gc)^2}\frac{R_1 q}{R_2^2 - R_1^2} + \frac{\lambda}{Gc}[C_1 k_1 \exp(k_1 x) + C_2 k_2 \exp(k_2 x)] + \frac{2R_1 q}{Gc(R_2^2 - R_1^2)}x. \tag{9}$$

Equations (8) and (9) were used to numerically study the effect of the thermal conductivity of the porous material λ, the mass flow rate of the coolant G, the rate of volumetric intrapore heat transfer h_v, and the channel width $\Delta = R_2 - R_1$ on the character of change of the mean temperatures of the porous material and coolant along the channel.

Figure 1 shows the distributions of the mean temperatures of the porous material $\overline{T}$ (curves 1-3) and coolant $\overline{t}$ (curves 4-6) obtained by means of Eqs. (8) and (9). The calculations were performed for the following values of the parameters describing the motion of a coolant in a channel with a porous insert: $t_0 = 273$°K, $R_1 = 0.0125$ m, c = 1.06 kJ/kg·K, $\Delta = 0.012$ m, $l = 0.1$ m, G = 1 kg/m²·sec, $h_v = 100$ kW/m³·K, q = 116.3 kJ/m²·sec.

To ensure agreement among the above parameters when we selected their numerical values, we used the relation $h_v = 0.1Gc/d_p$, which was obtained in [3]. Curves 1 and 4, 2 and 5, and 3 and 6 were constructed from the results of calculations performed for values of the thermal conductivity of the porous material λ equal to $1.75 \cdot 10^{-3}$, $1.75 \cdot 10^{-2}$, and 1.75×10^{-1} kW/m·K, respectively.

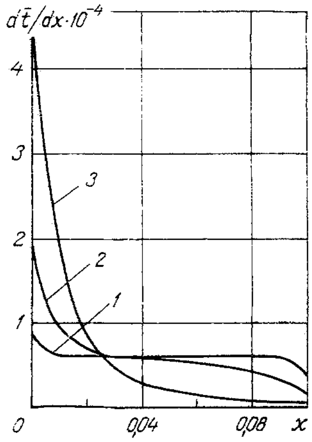

Fig. 2. Distribution of the longitudinal gradient of the mean temperature of the coolant along the channel. $d\bar{t}/dx$, K/m; x, m.

Figure 2 shows curves of the distribution of the gradient of the mean (across the channel) coolant temperature along a channel with a porous insert. Curves 1, 2, and 3 in Fig. 2 correspond to values of λ equal to $1.75 \cdot 10^{-3}$, $1.75 \cdot 10^{-2}$, and $1.75 \cdot 10^{-1}$ kW/m·K.

It can be seen from Figs. 1 and 2 that the distributions of the mean temperatures of the porous material and coolant depend considerably on the thermal conductivity of the porous material. For low-heat-conducting porous inserts ($\lambda \sim 10^{-3}$ kW/m·K), the change of these temperatures along the channel is quasilinear in character. This is evidence of the presence of an extensive (0.01 m < x < 0.09 m) section with a constant gradient of the mean coolant temperature (see curve 1 in Fig. 2). With an increase in the thermal conductivity of the porous material, the character of the distribution of the mean temperatures of the porous material and coolant changes and, as can be seen from Figs. 1 and 2, becomes quite nonlinear. Meanwhile, the maximum values of the mean temperature of the porous material and the absolute value of its longitudinal gradient decrease, while the absolute value of the longitudinal gradient of the mean coolant temperature increases.

Calculations of the distributions of the mean temperatures of the porous material and coolant performed for different values of the parameters G, h_V, and Δ but fixed values of λ showed that varying G, h_V, and Δ does not change the character of the distributions. Here, the distributions remain nonlinear in the case of low-heat-conducting porous inserts, while they are nonlinear in the case of inserts with a high thermal conductivity.

NOTATION

T, temperature of porous material; t, coolant temperature; x, axial coordinate; r, radial coordinate; λ, thermal conductivity of the porous material; h_V, rate of intrapore heat transfer; R_1, internal radius of the annular channel; R_2, external radius of the annular channel; G, mass flow rate of coolant; c, isobaric specific heat of coolant; t_0, coolant temperature at the channel inlet; $q_1(x)$, heat flux on the inside surface of the annular channel; $q_2(x)$, heat flux on the outside surface of the annular channel; d_p, particle diameter; Nu_V, Nusselt number; Pe, Peclet number.

LITERATURE CITED

1. D. Cox and R. Koloni, "Analysis of cooling efficiency for the case of coolant flow in a channel with a porous material," Trans. ASME Ser. C. Heat Transfer, 96, No. 3, 66-74 (1974).
2. V. M. Polyaev and V. A. Maiorov, "Heat transfer in a channel with a porous insert in a system of forced single-phase cooling," Izv. Vyssh. Uchebn. Zaved., Mashinostr., No. 7, 51-55 (1982).
3. O. G. Rasin, "Theoretical—experimental method of determining the thermal characteristics of channels with a porous filler," in: Energy and Mass Transfer Processes in Porous Media with Phase Transitions [in Russian], ITMO im. A. V. Lykova, Akad. Nauk Belorussian SSR, Minsk (1982), pp. 47-55.